A Frequency-Domain Mixer Model for Diffusion-Cooled Hot-Electron Bolometers

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***Abstract*— A new frequency domain hot-electron bolometer (HEB) mixer model is described, which takes into account the non-uniform temperature distribution found in superconducting diffusion-cooled devices. In this model the bolometer is discretized into a finite number of segments, and the non-linear large signal response to local oscillator and DC heating is found by an iterative algorithm. A frequency-domain coupling matrix is introduced to represent small-signal modulations around the large-signal values of temperatures and heat flows inside the device. The coupling matrix and a Norton representation of the mixer DC and intermediate frequency (IF) circuit allows the mixer conversion to be calculated.**

I. INTRODUCTION

Low-noise Superconducting Hot-Electron Bolometer (HEB) mixers operating at terahertz frequencies have attracted significant attention in recent years. This interest is primarily derived from the spectrometer needs of certain astrophysics missions, such as the National Aeronautics and Space Administration's (NASA) airborne observational platform SOFIA, and the European Space Agency's (ESA) spaceborne far-infrared telescope FIRST. Some of the key issues that justify the HEB development effort to these missions, besides the low-noise receiver performance, are the low local oscillator power requirements and the high (several gigahertz) intermediate frequency (IF) bandwidths that have been achieved in the last couple of years [1]-[6]. Analysis of superconducting hot-electron bolometer mixer performance has usually been done using lumped model approximations [7]-[9] that are adaptations of the theory developed for semiconductor devices [10,11]. The device is represented by a heat capacitance that is connected to a thermal bath via a heat conductance. This model does not accurately analyze diffusion-cooled bolometers, since the electron gas in such devices has a non-uniform temperature distribution. We have previously used a finite-difference time-domain simulation that models this temperature distribution, which resulted in a calculated conversion within 2 dB and a calculated device roll-off frequency within 25% of experimental values [12]. The calculation used a measured resistance-versus-temperature (RT) curve, but did not otherwise take into account any effects particular to superconductors, such as the coherence length. Another approach is described in [13], where the temperature profile is included in the model, and the film resistivity is assumed to change abruptly from zero to the normal state resistivity where the local temperature equals the film critical temperature.

In this paper we introduce a frequency-domain model for a diffusion-cooled HEB, that has two advantages compared to the time-domain simulation in [12]. The first is that no easy general rule exists for choosing the time-step that is used in the time-domain formulation, and as a result it must chosen small enough to result in rather lengthy calculations. The second advantage is that we expect that it will be easier in the future to include a representation of thermal fluctuation noise into the frequency-domain model. A potential drawback is that this frequency-domain model is inherently linear, so that mixer saturation effects cannot be accurately included in a straightforward way.

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II. LARGE-SIGNAL AND SMALL-SIGNAL DEVICE MODELS

The temperature distribution in a diffusion-cooled superconducting hot-electron bolometer is determined by the DC and high-frequency (RF) heating, and by internal heat conduction that is proportional to the temperature in accordance with the Wiedemann-Franz law:

$$\Phi = -G_0 \frac{T}{T_0} \nabla T \quad , \tag{1}$$

where Φ $(\frac{W}{m^2})$ is the heat flow , G_0 $(\frac{W}{m \cdot K})$ is the heat conductivity at temperature T_0, and T is the temperature. If the local oscillator (LO) frequency is higher than the superconducting gap frequency, the total RF heating power $2P_{rf}$ is uniformly distributed throughout the device. The one-dimensional differential equation for the steady-state temperature field in the bolometer can then be written in the form

$$-\frac{1}{2}\frac{G_0}{T_0}\frac{d\left(T^2\right)}{dx}=\frac{2P_{rf}}{w\cdot h\cdot L}x+\frac{I_0^2}{w^2h^2}\int_0^x \rho\left(x',T(x')\right)dx' \quad , \qquad (2)$$

where I_0 is the DC current, and w,h,L are the width, thickness and length of the device. ρ is the temperature dependent film resistivity, and x is the distance from the center of the bolometer. The differential equation is non-linear, and without substantially simplifying the expression for ρ, no general closed-form solution may be found. Therefore, a numerical model is used here, in which the device is discretized into a finite number of segments, each with its own temperature, resistance, heat conductance and heat capacitance, see Fig.1. This discretization is a mathematical tool only, and the segment size does not by itself represent any physical size parameter in the device, such as the coherence length ξ. The length of each segment must be chosen significantly smaller than any spatial variations in the physical parameters that are described by the model. Since the bolometer is symmetric, only half the device needs to be modeled.

The large-signal temperature profile inside the device is calculated using an iterative finite difference method. Each device bias point is represented by the temperature of the central segment, and the temperatures of the other segments are calculated using the heat flow equation with the high-frequency RF heating and the heating due to the DC current in each and every element taken into account. The RF heating is assumed to be uniform throughout the device, and the RF resistance of the device is assumed to be the equal to the DC normal resistance. The DC resistance of each segment is calculated using the local segment temperature and a predefined resistance-versus-temperature (RT) curve. The temperature of the end element is compared to the ambient temperature, and the DC current in the simulation is then adjusted repeatedly until the two agree. Each point on the device current-voltage (IV) curve is thus uniquely represented by the device center temperature at a fixed LO power level.

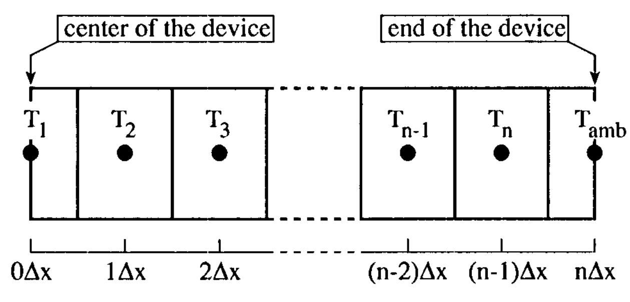

Fig. 1. The figure shows the discretization that is used in the large-signal model. Because of the symmetry, only half of the device needs to be considered. Tamb is the temperature of the normal metal contacts to the device. The total length of the device is L=2nΔx.

It is assumed that the LO and signal frequencies sufficiently exceed the device response time, and even the superconducting gap frequency, so that the effect of the signal can be seen as a time-dependent modulation of the uniform RF heating at the intermediate (IF) frequency. It is also assumed that this modulation is so small, that all resulting effects on physical parameters can be seen as linear perturbations of those parameters (the small-signal limit). The linearity makes it very straightforward to formulate a frequency domain representation for the heat dissipation and conduction processes in the device, and to join this representation with a model of the DC and IF embedding circuit. The temperature dependent heat capacity of each segment, as well as the Wiedemann-Franz heat conductance between adjacent elements, are calculated from the temperatures given by the large signal model. The entire bolometer is modeled as a thermal circuit using the same formalism as for small-signal electrical networks, Fig.2 .

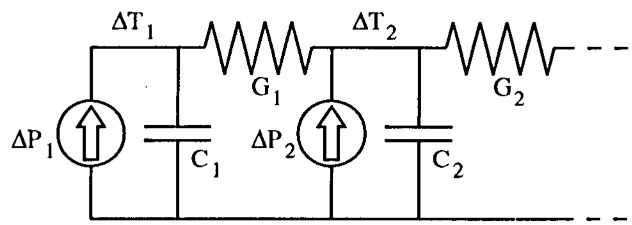

Fig. 2. Small-signal thermal model of the discretized bolometer. Element number 1 is at the center of the element. The element at the other end of the chain (not shown) is connected to the "thermal ground", since it is heat-sunk to the ambient temperature.

Heat dissipation in the segments is described as a number of independent current sources (ΔP_1, ΔP_2 et cetera), which affect the segment temperatures (ΔT_1, ΔT_2 et cetera) through a coupling matrix A,

$$[\Delta T_k]_k=\left[A_{kl}\right]_{kl}\cdot\left[\Delta P_l\right]_l \qquad , \qquad (3)$$

where brackets denote vectors and matrices. The matrix A can easily be calculated from its band-matrix inverse:

$$[A]^{-1}=\begin{bmatrix}(j\omega C_1+G_1) & (-G_1) & 0 & \dots \\ (-G_1) & (j\omega C_2+G_1+G_2) & (-G_2) & \dots \\ 0 & (-G_2) & \dots & \dots \\ \dots & \dots & \dots & et\ c.\end{bmatrix} \qquad (4)$$

III. CALCULATION OF CONVERSION GAIN

The mixer conversion can be calculated using the matrix description of the device. In the small-signal limit the heat dissipation in segment k can be written as

$$\Delta P_k=\Delta P_{rf,k}+\Delta P_{bias,k}=\Delta P_{rf,k}+2R_{0,k}I_0\Delta I+I_0^2\Delta R_k \quad , \qquad (5)$$

Where $\Delta P_{rf,k}$ is the modulation in RF heating due to the beating between signal and LO, and $\Delta P_{bias,k}$ is the heating by the DC/IF low-frequency current through the device. I_0 and ΔI are the large-signal (DC) and small-signal currents through the device. $R_{0,k}$ and ΔR_k are the large-signal and small-signal resistances of element k. The total small-signal modulation ΔR of the device resistance R is

$$\Delta R = \sum_k \Delta R_k \qquad , \tag{6}$$

where ΔR_k is the small-signal resistance of element k. With the expressions above for the heating and the coupling matrix A, ΔR_k can be expanded:

$$\Delta R_k = \frac{\partial R_k}{\partial T_k}\Delta T_k = \frac{\partial R_k}{\partial T_k}\sum_l A_{kl}\left(\Delta P_{rf,k} + 2R_{0,k}I_0\Delta I + I_0^2\Delta R_k\right) =$$
$$\frac{\partial R_k}{\partial T_k}\sum_l A_{kl}\Delta P_{rf,k} + \frac{\partial R_k}{\partial T_k}\cdot 2I_0\Delta I\sum_l A_{kl}R_{0,l} + \frac{\partial R_k}{\partial T_k}I_0^2\sum_l A_{kl}\Delta R_l \tag{7}$$

At this point a model of the DC/IF embedding circuit is needed. One of the simplest (and most useful) networks is the Norton equivalent current source shown in Fig.3 . It provides formulas for the DC bias current I_0, the device IF current ΔI and IF voltage ΔV:

$$I_0 = \frac{I_S R_S}{R_0 + R_S} \qquad , \tag{8}$$

$$\Delta I = -\frac{I_0}{R_0 + R_S}\cdot\Delta R \qquad , \tag{9}$$

$$\Delta V = -R_S\cdot\Delta I \qquad , \tag{10}$$

where I_S is the DC source current in Fig.XYZ. Since the device sees a passive IF output network, the IF current source value is 0. R_S is the impedance of the IF system.

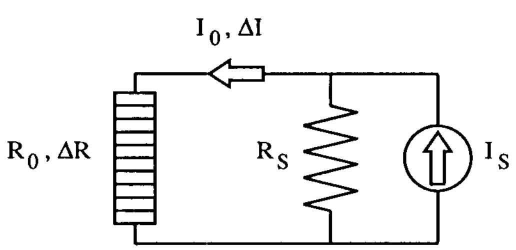

Fig. 3. The DC/IF embedding circuit is a Norton equivalent source at DC, and a passive load at the IF frequency.

The expression for ΔR_k above can now be rewritten as the matrix equation

$$\left[\frac{\partial R_k}{\partial T_k}\cdot\frac{2I_0^2}{R_0 + R_S}\cdot\left(\sum_m A_{km}R_{0,m}\right) + \frac{\partial R_k}{\partial T_k}I_0^2 A_{kl} - \delta_{kl}\right]_{kl}\cdot\left[\Delta R_l\right]_l =$$
$$\left[\frac{\partial R_k}{\partial T_k}\sum_l A_{kl}\Delta P_{rf,l}\right]_k \qquad , \tag{12}$$

where $\delta_{kl} = 1$ for $k = l$, $\delta_{kl} = 0$ for $k \neq l$. The equation above takes mismatch to the IF load into account (including self-heating effects). For simplicity, a response parameter E can be defined:

$$E = \frac{\Delta R}{\Delta P_{rf}} = \frac{\sum_k \Delta R_k}{\sum_k \Delta P_{rf,k}} \tag{13}$$

The time domain local oscillator and signal voltage over the bolometer can be written:

$$V_{LO}\cos\omega_{LO}t + V_S\cos\omega_S t \tag{14}$$

The modulation of the RF heating at the IF is

$$\Delta P_{rf} = \frac{V_{LO}V_S}{R_N} \tag{15}$$

so that the intermediate frequency voltage can be written:

$$\Delta V = \frac{R_S^2 I_S}{\left(R_0 + R_S\right)^2}\Delta R = \frac{R_S^2 I_S}{\left(R_0 + R_S\right)^2}\cdot E\cdot\frac{V_{LO}V_S}{R_N} \tag{16}$$

The conversion efficiency η is the ratio between the IF power coupled to the load R_S and the signal power coupled to the device:

$$\eta = 2\cdot R_S P_{rf}\left|\frac{I_0 E}{R_0 + R_S}\right|^2 \qquad . \tag{17}$$

IV. DISCUSSION

It should be noted that the theoretical model of bolometric mixing described in this paper is by no means complete. It does not treat in adequate detail effects that can occur in extremely short bridges, where the electron-electron scattering length can be comparable in size or greater than the device length. It also does not treat some relevant effects that are specific to superconductors, such as the coherence length ξ. A more complete mixer theory should also include thermal fluctuation noise, and we believe that our model can be extended to include this by introducing a random noise source into each of the device segments. Development of this noise model, as well as a numerical implementation of the theory in this paper are both underway, and will be presented in a future publication.

SUMMARY

We have developed a new frequency-domain theoretical mixer model for diffusion-cooled hot-electron bolometers, that uses a finite-difference method to determine the steady-state large signal parameters of the device, and a matrix equation formulation to calculate the small-signal conversion efficiency. In the small-signal model, the device is represented by a thermal circuit that is analogous to an electrical network. The circuit "components" are heat capacitances and heat conductances that are calculated from the large-signal model, and heat sources that represent dissipation of RF and low-frequency power. The thermal model is combined with an electrical circuit representation of the DC/IF embedding network, so that self-heating effects will be correctly included into the model.

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REFERENCES

[1] J. Kawamura, R. Blundell, C.-Y.E. Tong, D.C. Papa, T.R. Hunter, G. Gol'tsman, S. Cherednichenko, B. Voronov, E. Gershenzon, "First light with an 800 GHz phonon-cooled HEB mixer receiver", Proc. Ninth Int. Symp. Space Terahertz Tech., 17-19 March 1998, Jet Propulsion Laboratory, Pasadena, CA, pp. 35-43.

[2] B.S. Karasik, M.C. Gaidis, W.R. McGrath, B. Bumble, H.G. LeDuc, "Low noise in a diffusion-cooled hot-electron mixer at 2.5 THz", Appl. Phys. Lett. 71 (11), 15 September 1997, pp. 1567-1569.

[3] P.J. Burke, R.J. Schoelkopf, D.E. Prober, A. Skalare, W.R. McGrath, B. Bumble, H.G. LeDuc, "Length scaling of bandwidth and noise in hot-electron superconducting mixers", Appl. Phys. Lett. 68 (23), 3 June 1996, pp.3344-3346.

[4] B.S. Karasik, A. Skalare, R.A. Wyss, W.R. McGrath, B. Bumble, H.G. LeDuc, J.B. Barner, A.W. Kleinsasser, "Low-noise and wideband hot-electron superconductive mixers for THz frequencies", Proc. Sixth Int. Conf. Terahertz Electronics, 3-4 September 1998, University of Leeds, Leeds, United Kingdom.

[5] P. Yagoubov, M. Kroug, H. Merkel, E. Kollberg, G. Gol'tsman, A. Lipatov, S. Svechnikov, E. Gershenzon, "Quasioptical NbN phonon-cooled hot electron bolometric mixers with low optimal local oscillator power", Proc. Ninth Int. Symp. Space Terahertz Tech., 17-19 March 1998, Jet Propulsion Laboratory, Pasadena, CA, pp. 131-140.

[6] A. Skalare, W.R. McGrath, B. Bumble, H. LeDuc, "Measurements with a diffusion-cooled Nb hot-electron bolometer mixer at 1100 GHz", Proc. Ninth Int. Symp. Space Terahertz Tech., 17-19 March 1998, Jet Propulsion Laboratory, Pasadena, CA, pp. 115-120

[7] H. Ekström, B.S. Karasik, E.L. Kollberg, K.S. Yngvesson, "Conversion gain and noise of niobium superconducting hot-electron-mixers", IEEE Trans. Microwave Theory Tech., Vol. 43, No.4, April 1995, pp. 938-947

[8] P.J. Burke, "High frequency electron dynamics in thin film superconductors and applications to fast, sensitive THz detectors", Ph.D. dissertation, Yale Univ., New Haven, Connecticut, December 1997.

[9] H.F. Merkel, E.L.Kollberg, K.S.Yngvesson, "A large signal model for phonon-cooled hot-electron bolometric mixers for THz frequency applications,, Proc. Ninth Int. Symp. on Space Terahertz Technology, Jet Propulsion Laboratory, Pasadena, CA, 17-19 March 1998, pp.81-97

[10] F. Arams, C. Allen, B. Peyton, E. Sard, "Millimeter mixing and detection in bulk InSb", Proc. IEEE, Vol.54, pp.308-318, 1966

[11] J.C. Mather, "Bolometer noise: nonequilibrium theory", Appl. Opt. 21, pp.1125 (1982)

[12] A. Skalare, W.R. McGrath, B. Bumble, H.G. LeDuc, P.J. Burke, A.A. Verheijen, R.J. Schoelkopf, D.E. Prober, "Large bandwidth and low noise in a diffusion-cooled hot-electron bolometer mixer", Appl. Phys.Lett. 68 (11), 11 March 1996, pp.1558-1560.

[13] D.W. Floet, J.J.A. Baselmans, J.R. Gao, T.M. Klapwijk, "Resistive behaviour of Nb diffusion-Cooled hot-electron bolometer mixers", Proc. Ninth Int. Symp. on Space Terahertz Technology, Jet Propulsion Laboratory, Pasadena, CA, 17-19 March 1998, pp.63-72